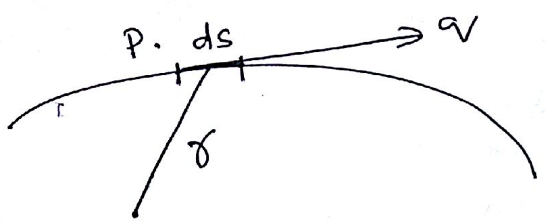
Lagrangian : individual fluid particle ①
Eulerian : at a point

Stream line : Stream line is a continuous line of flow drawn in the fluid so that ~~the~~ tangent at every point is the direction of ~~velocity~~ fluid velocity at the point at a given instant. The component of velocity at right angles to the streamlines is always zero. This shows that there is no flow across the streamline. Thus, a solid boundary is also a streamline.



Consider $\vec{ds}$ be an element of stream line passing through any point P having position vector $\vec{r}$ at certain instant of time.

Let $\vec{q}$ be the velocity vector at that point at the same instant. Since the direction of ds is same as direction of velocity.

i.e $\vec{ds} \parallel \vec{q}$.

$$\vec{ds} = \hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz.$$

$$\vec{q} = u\hat{i} + v\hat{j} + w\hat{k}$$

$$\vec{q} \times \vec{ds} = \begin{vmatrix} i & j & k \\ u & v & w \\ dx & dy & dz \end{vmatrix} = 0$$

$$= i\,[v\,dz - w\,dy] + j\,[w\,dx - u\,dz] + k\,[u\,dy - v\,dx] = 0$$

$w\,dy - v\,dz = 0 \Rightarrow \dfrac{dz}{w} = \dfrac{dy}{v}$ c = b

$u\,dz - w\,dx = 0 \Rightarrow \dfrac{dx}{u} = \dfrac{dz}{w}$ a = c

$v\,dx - u\,dy = 0 \Rightarrow \dfrac{dx}{u} = \dfrac{dy}{v}$ a = b

$\Rightarrow a = b = c$

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

Path line:

A curve described in space by a moving fluid element is known as its trajectory or path line.

If ~~p~~ $\vec{q}$ be the velocity and $\vec{r}(t)$ is a position vector ~~vector~~ ~~of a~~ point in a fluid

path ~~linde~~ line is

$$\boxed{\vec{q} = \frac{d\vec{r}}{dt}.}$$

$\vec{q} = u\hat{i} + v\hat{j} + w\hat{k}$

$\vec{r}(t) = \hat{i}\,x(t) + \hat{j}\,(y(t)) + \hat{k}\,z(t).$

$$u(x, y, z, t) = \frac{dx}{dt}.$$

$$v(x, y, z, t) = \frac{dy}{dt}.$$

$$w(x, y, z, t) = \frac{dz}{dt}.$$

$= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$

$= \frac{\partial x}{\partial x} - \frac{\partial y}{\partial y}$

$= 0.$

If it $: \boxed{\nabla \cdot q = 0}$

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$

$\frac{\partial u}{\partial x} = 1$

$\frac{\partial v}{\partial y} = -1$

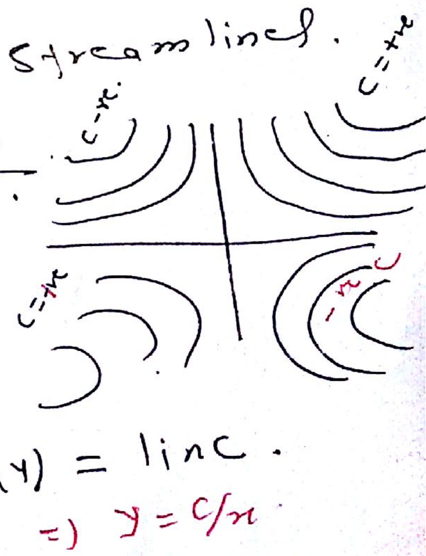
Q. Velocity vector $\vec{q}$ is given by

$$\bar{q} = \cancel{\;} x\hat{i} - y\hat{j}.$$

$$= u\hat{i} + v\hat{j}.$$

Determine the equation of stream lines. Also, ~~is~~ is it possible motion?

$$\vec{q} \times d\vec{s} = 0. \qquad \frac{dx}{u} = \frac{dy}{v}.$$



Sin

$$\frac{dx}{x} = \frac{dy}{-y}.$$

$y = \frac{c}{x}. \Rightarrow$

x +ve, c +ve, y +ve.

x −ve, c −ve, y +ve.

x −ve, c +ve, y −ve.

x +ve, c −ve, y −ve.

~~Integrating~~ it.

$$\ln(x) + \ln(y) = \ln c.$$

$$\underline{xy = c}. \Rightarrow y = c/x.$$

which represents that the stream lines are the rectangular hyperbola where c is an arbitrary constant.

2. The velocity $\vec{q}$ in a three dim. flow field for an incompressible fluid is given by $\vec{q} = 2x\hat{i} - y\hat{j} - z\hat{k}$. Is it a possible field? Determine the eqn. of streamline passing through (1, 1, 1).

The equation of continuity for an incompressible flow is given by

$$\vec{\nabla}\cdot\vec{q} = 0.$$

$$\nabla\cdot(2x\hat{i} - \hat{j}y - z\hat{k}) = 0.$$

$$= 2 - 1 - 1 = 0.$$

which shows that it is a possible field.

Eqn. of streamlines are given by

$$\frac{dx}{2x} = \frac{dy}{-y} = \frac{dz}{-z}.$$

(I) (II) (III)

(I) & (II)

$$\ln x + \ln(y^2) = \ln(A) \Rightarrow xy^2 = A$$

where A is integration constant.

(I) & (III)

$$\ln x + \ln(z^2) = \ln(B).$$

$$xz^2 = B.$$

where B is a constant.

At point (1, 1, 1) $A = 1$, $B = 1$.

Hence the required stream lines are

$\boxed{xy^2 = 1}$ $\boxed{xz^2 = 1}$

$q = 2x\hat{i} - y\hat{j} - z\hat{k}$

$q = u\hat{i} + v\hat{j} + w\hat{k}$

$u = 2x$

$v = -y$

$w = -z$

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$

$2 - 1 - 1 = 0.$

$\frac{dx}{u} = \frac{dy}{v} = \frac{dw}{w}$

Ex. The velocity field at a point in fluid is given as $\vec{q} = (x/t, y, 0)$. Obtain the path lines.

$$\vec{q} = \frac{d\vec{r}}{dt}.$$

when at $t = t_0$, $x = x_0$, $y = y_0$

$$u = \frac{x}{t}, \quad V = y \quad \& \quad w = 0.$$

$$\underbrace{\frac{dx}{dt} = \frac{x}{t}}_{\mathrm{I}}, \quad \underbrace{\frac{dy}{dt} = y}_{\mathrm{II}}; \quad \underbrace{\frac{dz}{dt} = 0}_{\mathrm{III}}.$$

$\vec{q} = u\hat{i} + v\hat{j} + w\hat{k}$

$$\ln(x) - \ln(t) = \ln(A).$$

$$x = At \quad -(1)$$

$$\ln(y) = t + \ln(B).$$

$$\boxed{y = \exp(t).B.}$$

$$z = \text{constant}. \quad -(3)$$

at $t = t_0$ $x = x_0$.

$$A = \frac{x_0}{t_0}.$$

at $t = t_0$ $y = y_0$.

$$B = y_0 \exp(-t_0)$$

Hence the path lines are.

Thus $$x = \frac{x_0 t}{t_0} \quad -(4)$$

$$y = y_0 \exp(t - t_0). \quad (5)$$

$$z = \text{constant}. \quad -(6)$$

(3)

Find the equation of the stream lines for the flow $\vec{q} = -i(3y^2) - j(6x)$ at a point (1, 1).

Stream lines:

prove it: $\nabla \cdot q = 0$

$\left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y}\right) \cdot q$

$$\frac{dx}{u} = \frac{dy}{v}.$$

$$\Rightarrow \frac{dx}{-3y^2} = \frac{dy}{-6x}.$$

$$+2x\,dx = y^2\,dy.$$

$$+x^2 = \frac{y^3}{3} + C \quad \text{at } 1, 1.$$

$$C = +1 - \frac{1}{3} = \frac{2}{3}.$$

$$x^2 = \frac{1}{3}y^3 + \frac{2}{3}. \quad \text{Ans.}$$

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$

$e^x ch(y) - e^x ch(y) = 0.$

(4) The velocity components in a two dim. flow field is given by

$u = e^x \cosh(y)$

$v = -e^x \sinh(y)$. find the eq. of streamlines.

$\nabla \cdot q = 0.$

$\int \frac{\cosh(y)}{\sinh(y)} dy.$

$\sinh(y) = z.$

$dz = \cosh(y)\,dy.$

$\frac{dz}{z} = \ln(z).$

$\frac{\partial u}{\partial x} = e^x ch(y)$

$\frac{\partial v}{\partial y} = -e^x ch(y)$

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$

$$\frac{dx}{u} = \frac{dy}{v}.$$

$$\frac{dx}{e^x\cosh(y)} = \frac{dy}{-e^x\sinh(y)}.$$

$$dx + \frac{\cosh(y)}{\sinh(y)}\,dy = 0.$$

Let $z = \sinh(y)$

$dz = \cosh(y)$

$\int \frac{dz}{z} = \ln(z)$

$$x + \ln(\sinh(y)) = \ln(C).$$

$\ln(C) - \ln(\sinh(y))$

$= x$

$\frac{C}{\sinh(y)} = e^x$

$C = e^x \sinh(y)$

$$\frac{C}{\sinh(y)} = \exp(x). \Rightarrow \boxed{\sinh(y) = C\exp(-x)}$$

Ex.

If $u = x^2 + y^2 + z^2$ and

$v = -xy - yz - zx$, determine the

velocity component w which will satisfy

Continuity eqn. for incompressible fluid.

$$\nabla \cdot \bar{V} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.$$

Q. 2 a.

$$\frac{\partial w}{\partial z} = -\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right]$$

$$= -\left[2x - (x + z) \right]$$

$$= \cancel{(x+} \quad -x + z.$$

Integ. w.r.t. z.

$$w = -xz + \frac{z^2}{2} + f(x, y, t).$$ O.K. 4b.20 hrs

This shows that the soln. is not unique.

$-x + z$

1

Prove that the flow is irrotational for a given velocity field

$$q = (-iy + jx)/x^2+y^2$$

. Calculate the circulation round a square with its corner at $(1,0), (2,0), (2,1)$ and $(1,1)$

Soln

The flow is irrotational if it satisfies the condition

$$\vec{\nabla} \times \vec{q} = 0.$$

$$\vec{\nabla} \times \vec{q} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -\frac{y}{x^2+y^2} & \frac{x}{x^2+y^2} & 0 \end{vmatrix}$$

$$\nabla \times q = k\left[\frac{y^2-x^2}{(x^2+y^2)^2} + \frac{x^2-y^2}{(x^2+y^2)^2}\right] = 0.$$

$\frac{\partial}{\partial y}\left(\frac{y}{x^2+y^2}\right) = \frac{x^2+y^2-2y^2}{(x^2+y^2)^2} = \frac{x^2-y^2}{x^2+y^2}$ cont

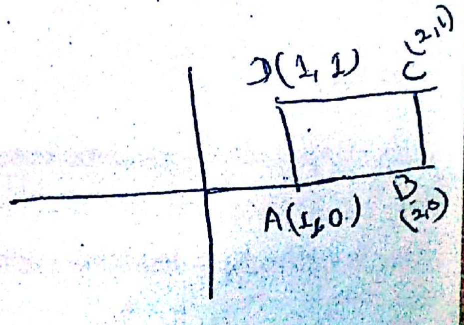
$\frac{\partial}{\partial x}\left(\frac{x}{x^2+y^2}\right) = \frac{x^2+y^2-x\cdot 2x}{x^2+y^2} = \frac{y^2-x^2}{x^2+y^2}$ cont

$\partial\left(\frac{y}{x^2+y^2}\right) = \frac{x^2+y^2-2y\cdot y}{(x^2+y^2)^2} = x^2-y^2$

$= \frac{\partial}{\partial x}\left(\frac{x}{x^2+y^2}\right) = \frac{x^2+y^2-x\cdot 2x}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$

It follows that $\nabla \times q = 0$ everywhere except at the origin.

The circulation Γ around a square is given by



$$\Gamma = \int \vec{q} \cdot d\vec{r}$$

$i \cdot j = 0$

$$= \int_A^B \vec{q} \cdot d\vec{r} + \int_B^C \vec{q} \cdot d\vec{r} + \int^D \vec{q} \cdot d\vec{r} + \int^A \vec{q} \cdot d\vec{r}.$$

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Since along BC $\quad x = 2 \quad dx = 0.$

$dr = i\,dx + j\,dy = j\,dy$

$$= \int_0^1 \frac{2\,dy}{4+y^2} = \tan^{-1}\left[\tfrac{1}{2}\right]. \quad \tan^{-1}(0) = 0$$

$\int \frac{1}{4+y^2}$

$\tan^{-1}(y/2)$

$\frac{1}{2}$

Now

$$\int_C^D \vec{q}\,d\vec{r} = \int_{x=2}^{1} \frac{(-iy + jx)}{x^2+y^2} \cdot i\,dx.$$

Since along CD $\quad y = 1 \Rightarrow dy = 0.$

$$\int_2^1 = -\int_1^2$$

$$= \int_1^2 \frac{dx}{x^2+1} = \tan^{-1}[2] - \tan^{-1}[1]$$

Also

$$\int_D^A \vec{q}\cdot d\vec{r} = \int_1^0 \frac{-iy + jx}{x^2+y^2}\; j\,dy$$

$\therefore \; x = 1 \quad dx = 0.$

$$= \int_1^0 \frac{dy}{1+y^2} = -\tan^{-1}[1].$$

Thus $\quad \Gamma = \tan^{-1}\left[\tfrac{1}{2}\right] + \tan^{-1}[2] - 2\tan^{-1}(1)$

$$= \tan^{-1}\left[\frac{\frac{1}{2}+2}{1-1}\right] - 2\cdot \pi/4$$

$\tan^{-1}[A] + \tan^{-1}(B) = \tan^{-1}\left[\frac{A+B}{1-A\cdot B}\right]$

$$= \tan^{-1}(\infty) - \pi/2.$$

$$= \pi/2 - \pi/2 = 0.$$

Div. grad ϕ.

$$\text{grad } \phi = \frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} \hat{j} + \frac{\partial \phi}{\partial z} \hat{k}$$

$$\nabla \cdot \nabla \phi = \left(i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right) \cdot \left(\frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} j + \frac{\partial \phi}{\partial z} \hat{k} \right)$$

$$= \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = \nabla^2 \phi.$$

LAGRANGIAN METHOD.

The Lagrangian method traces the progress of the individual fluid particles in their movement.

Let at time t_0 $(t_0 = 0)$ the position ~~of the particle~~ is $\vec{\xi}(a, b, c)$ and the position vector of the same particle at ~~$t > 0$ some~~ a later time ~~$r(x, y, z)$~~ at $t > 0$ is $\vec{r}(x(t), y(t), z(t))$.

The velocity is $\frac{d\vec{r}}{dt}$ & acc. is $\frac{d^2\vec{r}}{dt^2}$.

$\vec{V} = \frac{d\vec{r}}{dt}$

$ui + vj + wk = i\frac{dx}{dt} + j\frac{dy}{dt} + k\frac{dz}{dt}$

$u = \frac{dx}{dt}$, $v = \frac{dy}{dt}$, $w = \frac{dz}{dt}$

Eulerian Method.

In this method the individual particles are not identified but attention is paid to a point of the fluid. We observe what happens at a specific points that are fixed in space within the flow field as different particles pass through them ~~during~~ in the course of time.

Accordingly, a complete decreiption of the flow involves an instantaneous picture of velocities at all point's in the field.

1 dim. flow
2. dim flow
3. dim flow.

Ex. For a two dim. flow the velocities at a point in the fluid may be expressed in the Eulerian co-ordinates by

$u = x + y + 2t$ and $v = 2y + t$.

Determine the Lagrange co-ordinates as a function of the initial position x_0 & y_0 and the time $t = t_0 = 0$. $x = x_0$ & $y = y_0$.

Thus $u = \frac{dx}{dt}$ & $v = \frac{dy}{dt}$.

~~Thus~~ then. $\frac{dx}{dt} = x + y + 2t$ — (1)

$\frac{dy}{dt} = 2y + t$. — (2).

$\frac{dy}{dx} + P(x)y = Q(x)$.
$I.F = e^{\int P dx}$.
$y\, I.F = \int Q(x) I.F + C$.

$\frac{dy}{dt} - 2y = t$.

$y e^{-2t} = \int t e^{-2t} dt + C$

$-t \frac{e^{-2t}}{2} + \int \frac{e^{-2t}}{2} dt$.
$-\frac{te^{-2t}}{2} - \frac{e^{-2t}}{4}$
$-\frac{e^{-2t}}{4}(2t+1)$

$y e^{-2t} = C - \frac{e^{-2t}}{4}(2t+1)$

$y(t) = Ce^{2t} - \frac{1}{4}(2t+1)$. — (3)

Use (3) in (1).

$\frac{dx}{dt} - x = Ce^{2t} - \frac{1}{4}(2t+1) + 2t$.

$\frac{dx}{dt} - x = Ce^{2t} + \frac{1}{4}(6t-1)$

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$$x\,e^{-t} = c\int e^{t}\,dt + \frac{1}{4}\int (6t-1)e^{-t}\,dt + B.$$

$$x(t) = c\,e^{2t} + B\,e^{t} - \frac{1}{4}[6t+5].$$

$$6\int t\,e^{-t} = -t\,e^{-t} - e^{-t}.$$

$$-\int e^{-t}\,dt = e^{-t}.$$

$$\text{Thus} \int (6t-1)\,e^{-t}\,dt = \frac{1}{4}[-6t-5]$$

$$= -\frac{1}{4}[6t+5].$$

The Constant A & B are determined from the initial Conditions $x = x_0$ & $y = y_0$ at $t = t_0 = 0$.

Lagrangian

$$c = y_0 + \frac{1}{4}.$$

$$x_0 = y_0 + \frac{1}{4} + B - \frac{5}{4}$$

$$B = x_0 - y_0 + 1.$$

$$\text{Thus } x = \left(y_0 + \frac{1}{4}\right)e^{2t} + (x_0 - y_0 + 1)\,e^{t} - \frac{1}{4}(6t+5).$$

$$y(t) = \left(y_0 + \frac{1}{4}\right)e^{2t} - \frac{1}{4}(2t+1).$$

Ex: The velocity components for a two-dimensional flow system can be given in the Eularian system by

$$u = 2x + 2y + 3t, \quad v = x + y + \frac{1}{2}t.$$

Find the displacement of a fluid particle in the Lagrangian system.

$$u = \frac{dx}{dt} = 2x + 2y + 3t \Rightarrow \frac{dx}{dt} - 2x = 2y + 3t \quad -(1)$$

$$v = \frac{dy}{dt} = x + y + \frac{1}{2}t \Rightarrow \frac{dy}{dt} - y = x + \frac{1}{2}t \quad -(2)$$

For elimination of x

operating $\left(\frac{d}{dt} - 2\right)$ on (2) and add it (1) & (2).

elimination of y: operate $\left(\frac{d}{dt} - 1\right)$ on (1)

$\left(\frac{d}{dt} - 1\right)\left(\frac{dx}{dt} - 2x\right)$

$= 2\left[x + \frac{1}{2}t\right] = 3t + 3.$

$$\frac{dx}{dt} - 2x = 2y + 3t.$$

$$-\left[\frac{dx}{dt} - 2x\right] + \left(\frac{d}{dt} - 2\right)\left(\frac{dy}{dt} - y\right) = \left(\frac{d}{dt} - 2\right)\left(\frac{1}{2}t\right).$$

$$\left(\frac{d}{dt} - 2\right)\left(\frac{dy}{dt} - y\right) = 2y + 3t + \frac{1}{2} - t$$

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} - 2\frac{dy}{dt} + 2y = 2y + 2t + \frac{1}{2}.$$

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} = 2t + \frac{1}{2}.$$

$\frac{dy}{dx} + P(x) = Q(x)$

$y \cdot I \cdot F = \int I \cdot F \cdot Q(x)\,dx + c.$

$$\frac{dy}{dt} - 3y = \cancel{2}\frac{t^2}{\cancel{2}} + \frac{1}{2}t + A.$$

$$\frac{dy}{dt} - 3y = t^2 + \frac{1}{2}t + A.$$

$$I \cdot F = e^{-3t}.$$

$$y e^{-3t} = \int\left[\left(t^2 + \frac{1}{2}t\right)e^{-3t} + A e^{-3t}\right]dt + B.$$

$$\int t^2 e^{-3t}\,dt = t^2\frac{e^{-3t}}{-3} + \frac{1}{3}\int 2t e^{-3t}\,dt.$$

$$= \frac{t^2 e^{-3t}}{-3} + \frac{2}{3}\left[t\frac{e^{-3t}}{-3} + \int e^{-3t}\,dt\right].$$

$$= \frac{t^2 e^{-3t}}{-3} - \frac{2}{9}t e^{-3t} - \frac{2}{9}e^{-3t}.$$

$$\int \frac{1}{2} t e^{-3t}\,dt = \frac{1}{2}\left[\frac{t e^{-3t}}{-3} + \frac{1}{3}\int e^{-3t}\,dt\right].$$

$$= \frac{1}{2}\left[-\frac{t e^{-3t}}{3} - \frac{1}{9}e^{-3t}\right].$$

$3\left|\frac{6, 9}{2, 3}\right.$

$\frac{-6-\cancel{4}}{18}$

$$\text{Thus } y(t) = A_1 + B\exp(3t) - \frac{7}{18}t - \frac{1}{3}t^2.$$

$$\text{where } A_1 = \frac{A}{-3} - \frac{2}{9} - \frac{1}{18} \Rightarrow \frac{4}{18} - \frac{1}{18} = \frac{3}{18} = \frac{1}{6}.$$

Since Usip (2)

$$\frac{dy}{dt} - y = x + \frac{1}{2}t \Rightarrow x = \frac{dy}{dt} - y - \frac{1}{2}t.$$

$$\cancel{x = -\frac{1}{2}t - \frac{dy}{dt} + y}$$

$$\cancel{= \frac{1}{2}t + A_1 + B\exp(3t) - \frac{7}{18}t - \frac{1}{3}t^2}$$

$$\cancel{- \left[3B\exp(3t) - \frac{7}{18} - \frac{2}{3}t\right]}.$$

$$\therefore x = -\frac{1}{2}t - \left[A_1 + B\exp(3t) - \frac{7}{18}t - \frac{1}{3}t^2\right]$$

$$+ \left[3B\exp(3t) - \frac{7}{18} - \frac{2}{3}t\right].$$

$$-\frac{1}{2} - \frac{2}{3} = \frac{-3-4}{6} = -\frac{7}{6} + \frac{7}{18}$$

$$x = -A_1 + 2B\exp(3t) + \frac{1}{3}t^2 - \frac{7}{9}t - \frac{7}{18}$$

$$2 \,|\, 6, 18 \quad 3 \quad 4 \quad \frac{-28+21}{24}$$

The constant A_1 & B are obtained by

asp

$x = x_0$ & $y = y_0$ at $t = t_0 = 0$

$$2 \,|\, 6, 18 \quad 3 \,|\, 3 \quad 9 \quad 1 \quad 3. \quad \frac{-21+7}{18} = -\frac{7}{9}$$

$$x_0 = -A_1 + 2B - \frac{7}{18}$$

$$y_0 = \boxed{A_1 + B} \Rightarrow A_1 + B$$

Add $x_0 + y_0 = 3B - \frac{7}{18}$

$$B = \frac{1}{3}\left[x_0 + y_0 + \frac{7}{18}\right].$$

$$A_1 = y_0 - B$$

$$A_1 = y_0 - \frac{1}{3}\left[x_0 + y_0 + \frac{7}{18}\right]$$

$$\cancel{A_1 = x_0}$$

$$= -\frac{1}{3}\left[x_0 - 2y_0 + \frac{7}{18}\right]$$

$$\cancel{A_1 = -x_0 + 2B - \frac{7}{18}}$$

$$\cancel{A_1 = -x_0 - \frac{2}{3}x_0 - \frac{2}{3}y_0 - \frac{7\times 2}{3\times 18} - \frac{7}{18}}$$

$$y = -\frac{1}{3}\left[x_0 - 2y_0 + \frac{7}{18}\right] + \frac{1}{3}\left[x_0 + y_0 + \frac{7}{18}\right]\exp(3t) - \frac{7}{18}t - \frac{1}{3}t^2.$$

$$x = \frac{1}{3}\left[x_0 - 2y_0 + \frac{7}{18}\right] + \frac{2}{3}\left[x_0 + y_0 + \frac{7}{18}\right]\exp(3t) + \frac{1}{3}t^2 - \frac{7}{9}t - \frac{7}{18}.$$

Ans

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